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THE SCALAR MESON PUZZLE BEYOND BCS

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We present a new perspective [1] on the scalar meson puzzle from spontaneous breaking of chiral symmetry beyond BCS. We find that going beyond BCS does not produce a systematic shift of the hadron spectrum. We also show that coupled channels reduce the breaking of chiral symmetry, with the same Feynman diagrams that appear in the coupling of a scalar meson to a pair of pseudoscalar mesons. With a Lattice QCD inspired quark-quark interaction, we find that the groundstate $I = 0$, ${}^3P_0$ $q\bar{q}$ meson is the $f_0(980)$ with a partial decay width of $40 MeV$. We also find a 30% reduction of the chiral condensate due to coupled channels.

1 A link from QCD to Hadronic Physics

We first integrate formally the gluons from the QCD action, and get an action of Dirac quarks which interact via cumulants of gluons [2]. This is the cumulant expansion,

$$\mathcal{H}_{int} = \frac{1}{2}\int d^4x\, d^4y\, \overline{\psi}(x)\gamma^\mu \frac{\lambda^a}{2}\psi(x)\; g^2\langle A^a_\mu(x)A^b_\nu(y)\rangle\; \overline{\psi}(y)\gamma^\nu\frac{\lambda^b}{2}\psi(y)\; + \;\cdots \tag{1}$$

The first cumulant, of two gluons, can be evaluated in the modified coordinate gauge,

$$\langle A^a_\mu(x)A^b_\nu(y)\rangle = x^k y^l \int_0^1 d\alpha d\beta \alpha^{n(\mu)}\beta^{n(\nu)}\langle F^a_{k\mu}(x_0,\alpha\mathbf{x})F^b_{l\nu}(y_0,\beta\mathbf{y})\rangle \tag{2}$$

as a function of the non local gluon condensate which can be evaluated on the lattice [3,4]. Other links from the Lattice to hadronic physics [5,6] were presented at this conference. If we use the approximation of including only the first gluon cumulant in the Bethe-Salpeter kernel, we

get a relativistic quark model, where the bare hadrons are described by a ladder of a quarks (antiquarks). Moreover, from the cumulant expansion we obtain a chiral invariant interaction.

Hadronic physics has 3 crucial properties that we now must recover from this quark model. Confinement is included from the onset in the quark-quark interaction. Spontaneous chiral symmetry breaking is achieved when the mass gap equation for the quark is solved. Strong interactions are finally recovered when the couplings of bare hadrons are included.

2 The BCS level

The BCS level corresponds to the minimal use of the interaction both in the mass gap equation and in the Bethe Salpeter equation [1].

2.1 A microscopic quark-quark interaction.

The calculation on the lattice [4] of the full non local gluon correlator lends support [2] for the picture of a simplified instantaneous model,

$$g^2 \langle A^a_\mu(x) A^b_\nu(y) \rangle \simeq \frac{-3}{4} \delta_{ab} g_{\mu\nu} \left\{ g_{\mu 0} \left[K_0^3 (\mathbf{x}-\mathbf{y})^2 - U \right] + a g_{\mu i} k_0^3 (\mathbf{x}-\mathbf{y})^2 \right\} \delta\left(x^0 - y^0\right) \quad . \tag{3}$$

In this interaction, a single scale K_0 is independent and needs to be fixed with the spectroscopy. The best fit is obtained [7] with $K_0 = 330 MeV$. The remaining parameters are determined from confinement ($U \to \infty$) and Lorentz invariance of the π ($a = -.18$). This single scale of hadronic physics can be measured for instance in the wave-functions of vector mesons [8].

2.2 The mass gap equation and constituent quarks

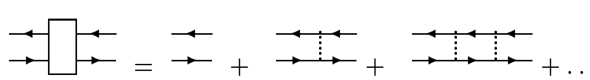
The mass gap equation is the Schwinger-Dyson equation, where V is the Fourier transform of the first cumulant,

$$\mathcal{S}^{-1}(p) = \mathcal{S}_0^{-1}(p) - \Sigma \ , \quad \Sigma = \int \frac{d^4 q}{(2\pi)^4} \gamma^\mu \frac{\lambda^a}{2} \mathcal{S}(q) V_{\mu\nu}(p-q) \gamma^\nu \frac{\lambda^b}{2} \quad . \tag{4}$$

For the interaction (3) several solutions for the Dirac quark propagator exist [7,9]. The solution for the dynamical (constituent) quark mass m which minimizes the vacuum energy density is illustrated in Fig. 1. We also find that the energy density of the vacuum is infinite and this might be relevant for gravitation [10].

2.3 The Bethe Salpeter equation and bare mesons

The boundstates appear as poles in the ladder,



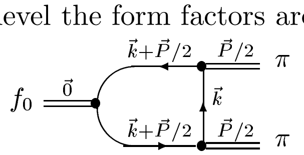
(5)

The full hadron spectrum can be computed [7]. In particular we find $M_{f_0} \simeq 1\, GeV$ and a massless π in the chiral limit.

2.4 Coupled Channels

The coupled channel contribution to the spectrum is evaluated with the mesonic form factors. At the quark level the form factors are computed with the overlaps,

$F(P) \quad =$



, (6)

where the blobs represent the vertex Bethe Salpeter amplitudes.

2.5 The scalar decay width

We finally find for the scalar meson,

$$\Gamma_{f_0\to\pi\pi} = 2iIm[M_{f_0}] = \frac{1}{4\pi} Q M_{f_0} \, |F(Q)|^2 \simeq 40MeV \ , \quad Q = \frac{1}{2}\sqrt{M_{f_0}^2 - 4M_\pi^2} \ . \tag{7}$$

3 Going beyond BCS

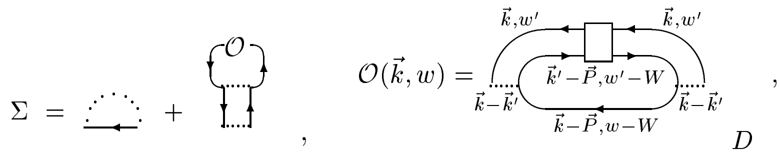
However, when the coupled channels are included in the boundstate equations, we should go beyond the BCS mass gap equation. Otherwise the π would get a negative contribution from the coupled channels, and it would have a paradoxal negative mass [1].

3.1 New mass gap equation

Naively, when the coupled channels in the Bethe Salpeter equation for the vertex, only the first line of eq.(10) is considered. Using the vector Ward Identity,

$$i(p_\mu - p'_\mu)\mathcal{S}(p)\Gamma^\mu(p,p')\mathcal{S}(p') = \mathcal{S}(p) - \mathcal{S}(p') \tag{8}$$

we replace the vertex by propagators and get the new quark self energy Σ,

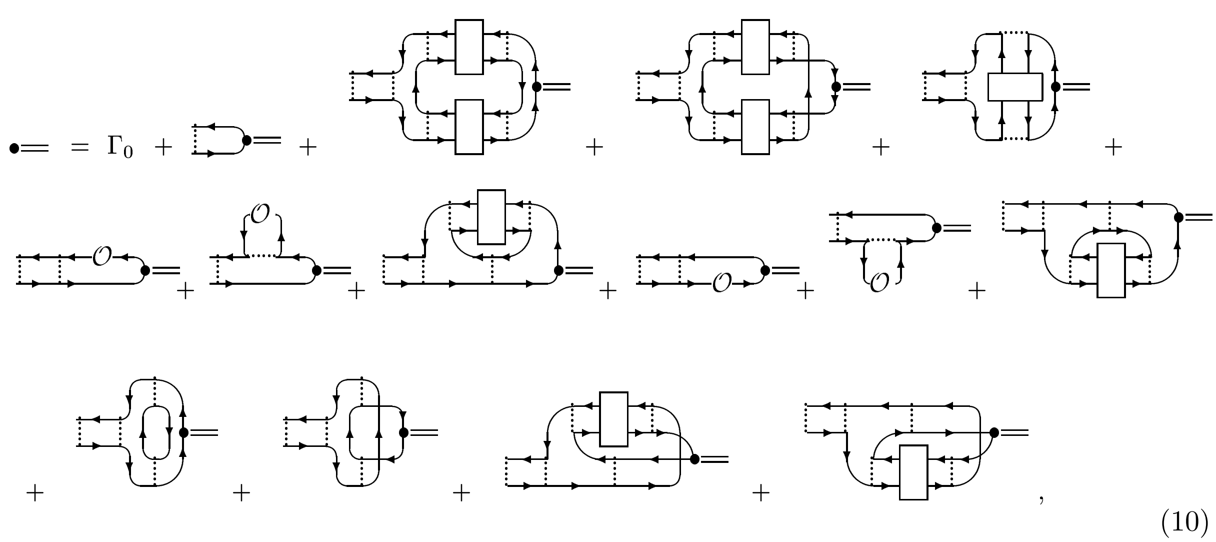


(9)

It turns out that a pseudoscalar loop and the coupling $F(p)$ of a scalar to a pair of pseudoscalars dominate the new coupled channel term. This opposes to the breaking of chiral symmetry. We show in Fig.[1] that the chiral angle which measures the extent of spontaneous breaking of chiral symmetry decreases.

3.2 New Bethe Salpeter equation

Inversely the new Bethe Salpeter equation, consistent with the new mass gap equation, is derived from eq.(9) with the vector Ward Identity (8). Replacing a propagator by a vertex in all possible combinations we get,



(10)

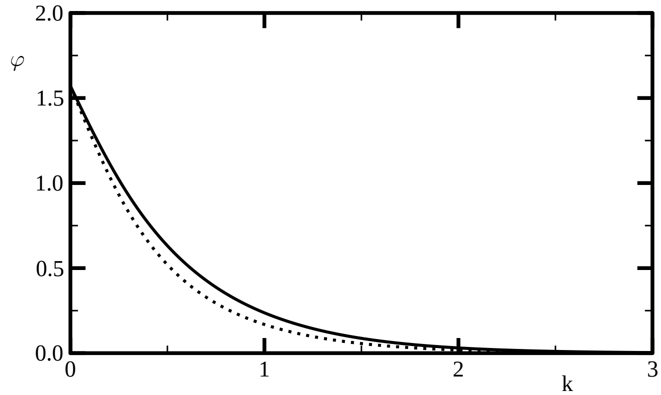


Figure 1: We show with a solid line the BCS chiral angle $\varphi = \arctan [m(k)/k]$ in dimensionless units of $K_0 = 1$. We also represent with a doted line the chiral angle that we obtain going beyond BCS.

where in the first line we include the BCS terms and the terms which are included in naive coupled channel studies. The BCS terms are responsible for most of the real mass of the mesons. The naive coupled channel terms yield the imaginary mass (width) of the mesons. They also produce a real contribution to the mass of mesons, however this is canceled by the remaining coupled channel terms, of the second and third lines in eq. (10), which readjust the spectrum in order to recover a vanishing mass for the pion in the chiral limit.

4 Conclusions

In this model we find that $f_0(980)$ and $a_0(980)$ are the light $q\bar{q}$ scalars. We also find that the quark $\langle\psi\bar{\psi}\rangle$ condensate is reduced 5% $\rightarrow$ 50% due to coupled channels.

We also prove general model independent results. The systematic real mass shift of the spectrum which might be due to coupled channels cancels out. We also show that there is a negative feedback: the coupling $f_0 \rightarrow \pi\pi$ suppresses chiral symmetry breaking and chiral symmetry breaking suppresses the coupling $f_0 \rightarrow \pi\pi$.

Presently the link to QCD is being studied with N. Brambilla, J.E. Ribeiro and A. Vairo, and similar techniques are now being applied to the nucleon with S. Cotanch, Felipe, R. Fernandes, J.E. Ribeiro and E. Swanson.

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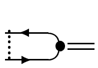
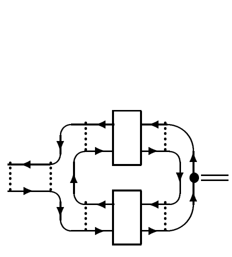
 $= \Gamma_0 +$  $+$  $\cdots$